{A Mathematical View: Linguistic Comprehension, Generation and Reasoning as Hypergraph Transformation}

The work presented here has been carried out primarily from a pragmatic computational linguistic perspective. The goal has been to explore some specific hypotheses regarding the intersection of linguistic phenomena and computational systems, in a manner with practical implications for the construction of natural language processing systems such as intelligent dialogue systems. However, it is also interesting to view the structures and algorithms we have developed in a broader mathematical context. In this brief chapter we present a mathematical perspective on the computational linguistics work reported here, utilizing the language of category theory.

我们的研究目标一直是探讨有关的语言现象和计算系统之间的交集，从而构建具有实用价值的自然语言处理系统，如智能对话系统。后面章节会通过介绍具体实现算法及相关应用来从语用计算语言学角度阐述本文的研究的可行性。本节主要从更广泛的数学角度，结合语言的范畴论来阐述其理论可行性。

\subsection{Basics of Category Theory}{范畴论的基础知识}

Category theory \cite{LawvereSchanuel97} is used to formalize mathematical structure and its concepts as a collection of objects and arrows (also called morphisms). A category has two basic properties: the ability to compose the arrows associatively and the existence of an identity arrow for each object. This simple set-up has a remarkable mathematical power and lends insight into essentially every domain of mathematical inquiry.

范畴论 \cite{LawvereSchanuel97}常被用于形式化各种数学结构中的共同特性，将这些数学结构的概念形式化成一组组对象和箭头（也称为态射）。一个范畴包括两个基本属性：对象之间的箭头可以复合 ；每个对象有一个标识箭头指向自己。对象和箭头可以是抽象的任何类型，这个简单的结构安排有着非凡的数学能力，使其能被用在探索各个领域的数学理论基础。

More formally, a category consists of:

\begin{itemize}

\item A class ob(C), whose elements are called objects;

\item A class hom(C), whose elements are called morphisms or maps or arrows. The expression hom(a, b) denotes the class of all morphisms from a to b.

\item A binary operation $\circ$, called composition of morphisms, such that for any three objects a, b, and c, we have $hom(b, c) ? hom(a, b) \rightarrow hom(a, c$). The composition of $f : a \rightarrow b$ and $g : b \rightarrow c$ is written as $g \circ f$ or $gf$, and is governed by two axioms:

\begin{itemize}

\item Associativity: If $f : a \rightarrow b$, $g : b \rightarrow c$ and $h : c \rightarrow d$ then $h \circ (g \circ f) = (h \circ g) \circ f$, and

\item Identity: For every object x, there exists a morphism $1x : x \rightarrow x$ called the identity morphism for x, such that for every morphism $f : a \rightarrow b$, we have $1b \circ f = f = f \circ 1a$.

\end{itemize}

\end{itemize}

Any directed graph generates a small category: the objects are the vertices of the graph, and the morphisms are the paths in the graph (augmented with loops as needed) where composition of morphisms is concatenation of paths. Such a category is called the free category generated by the graph.

\subsection{A Category-Theoretic View of Linguistic Hypergraph Transformations}

We now explain how the formalism of category theory may be used as a general framework for formalizing and interpreting the diverse linguistic comprehension, generation and reasoning algorithms described in these pages. The key to this formalization is the OpenCog Atomspace which serves as the common representational framework for these various algorithms.

Let us introduce the term ``t-Atom'' to refer to a triple (Atom, time interval, Atom-state). In most cases the state of an Atom consists of its TruthValue and its AttentionValue, plus unchanging aspects such as its type and is name.

A 't-Atomspace' associated with an OpenCog system, over a certain interval $T$ of time, consists of the set of t-Atoms $(A,I,S)$ so that

\begin{itemize}

\item A has existed within the OpenCog system's Atomspace at some time point $t \in T$

\item I is an interval between two changes of the state of Atom $A$

\item S is the state of Atom A during interval I

\end{itemize}

Given a t-Atomspace, one can form an 'activity graph' whose nodes are sets of tAtoms, and whose links record activities of cognitive processes that transform the graph. A link from node $N\_1$ to node $N\_2$ indicates that a cognitive process has taken $N\_1$ as input and given $N\_2$ as output.

Paths through the activity graph result from chaining together cognitive processes. In category-theoretic terms, the objects here are tAtom-sets, and the morphisms are paths through the activity graph indicating chains of cognitive activity.

For instance:

\begin{itemize}

\item The link parser transforms sets of AdjacencyLinks between WordNodes, into sets of relationships indicating link parser links between these WordNodes.

\item RelEx transforms sets of link parser relationships into sets of relationships indicating semantic relationships.

\item RelEx2Logic transforms sets of relationships output by RelEx, into sets of more abstract relationships more suitable for PLN inference.

\item PLN inference maps Atom-sets into Atom-sets, transforming knowledge extracted from natural language into different forms as suitable for connecting with other knowledge of various sorts.

\item Fuzzy matching, an approach to natural language question-answering (an important ingredient to intelligent dialogue, as described in Chapter \ref{chap:dialogue}) maps an Atom-set into another Atom-set judged as a near match.

\item Microplanning transforms a set $S$ of relationships, intended for articulation, into a new set consisting of a partitioning of $S$ into a subset $S\_1$ to be sent to SuReal for articulation, and a subset $S\_2$ saved for likely articulation in the near future.

\item Surface realization transforms a set of semantic relationships into a set of link parser relationships; and then transforms a set of link parser relationships into a set of AdjacencyLinks between WordNodes.

\end{itemize}

The comprehension, reasoning/matching and generation processes may thus be viewed very broadly as three morphisms that can be chained together to form a single morphism corresponding to an instance of linguistic response.

The quality of a response may be quantified by assigning a ``cost'' to each link in the activity graph -- cost being viewed as the opposite of confidence. Lower confidence in an instance of comprehension, generation or reasoning results in a higher cost to that instance. The objective of a dialogue system may be viewed as the provision of responses that meet pragmatic goals with the minimum cost in this sense.

A very high level mathematical view of this nature has limited practical utility, in the sense that it does not provide much guidance on which sorts of specific transformation rules are most useful in the context of the details of a particular language like English. However, it does provide a broader perspective, allowing one to step back from the linguistic particulars and get a clear sense of what sort of operations are being undertaken. The presentation of comprehension, generation and reasoning as a collection of hypergraph transformations, performed in appropriate sequence, makes clear the role that these linguistic operations play in the broader scope of cognition.